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Applications of Laplace Transform in Successive Radioactive Decay of Nucleus

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Abstract: **In this paper, we will discuss the applications of Laplace Transform in solving the differential equations related to successive radioactive decay of parent nucleus which is important subject in nuclear physics. Also the equations for calculating the daughter and grand- daughter nucleus are given.**

Keywords: **Laplace transforms method, Radioactive decay, parent nucleus, grand-daughter nucleus.**

1. INTRODUCTION

The Laplace Transform is a widely used integral transform in mathematics with many applications in science and engineering. The Laplace Transform can be interpreted as transformation from time domain where input and outputs are functions of time to the frequency domain where inputs and outputs are functions of complex angular frequency. The concept of Laplace transforms are applied in science and technology such as electric circuit analysis, communication engineering, control engineering and Nuclear physics as well as in solving differential equations in quantum mechanics [1,2,3,4]. In this paper the solutions of differential equations in successive radioactive decay of nucleus using Laplace transformations are given in details.

2. FORMULATION OF DIFFERENTIAL EQUATIONS IN SUCCESSIVE RADIOACTIVE DECAY OF NUCLEUS

When the parent nucleus disintegrates into a daughter nucleus, the daughter nucleus may be radioactive, then the daughter nucleus disintegrates into a granddaughter nucleus, and so the process continues until it finally reaches a stable nucleus, and the process is known as successive radioactive disintegration.

For example, the nucleus of radium 226 with a half-life of $1.6 \times 10^3 y$ disintegrates into radon 222 and the latter, with a half-life of 3.82 days, disintegrates into a polonium-218 nucleus, which is also radioactive, with a half-life of 3.05 minutes, and so the process continues until it finally reaches the stable lead-208 nucleus.

The purpose of studying successive decay is to know the number of atoms or nuclei in each member of this chain. So if we symbolize the number of atoms of the parent elements in time t by the N_1 and the decay constant for it is λ_1, the number of atoms of the daughter nucleus N_2, which in turn is active, and the decay constant for it is λ_2 the number of atoms of the granddaughter nucleus N_3, which is a stable element. If the boundary condition for all atoms or nucleus at time t=0 is $N_1 = N_1(0), N_2 = 0, N_3 = 0$

If we take into account that the rate of decay of the mother atoms is exactly equal to the rate of formation of the daughter atoms, and that the rate of decay of the daughter atoms is equal to the rate of formation of the granddaughter atoms, then the whole process can be expressed by the following three equations:

$$\frac{dN_1}{dt} = -\lambda_1 N_1 \qquad (1-1)$$

$$\frac{dN_2}{dt} = \lambda_1 N_1 - \lambda_2 N_2 \qquad (1-2)$$

$$\frac{dN_3}{dt} = -\lambda_2 N_2 \qquad (1-3)$$

The relationship(1-1) determines the rate of disintegration for the parent atoms, according to the Basic Law. As for the equation(1-2), it means that the daughter atoms are formed at a rate $\lambda_1 N_1$ and disintegrate at a rate $\lambda_2 N_2$, while the equation (1-3) determines the rate of formation of the granddaughter atoms N_3. [5]

By solving the set of equations(1-1,1-2,1-3), the number of atoms of each type of the three members of the chain N_1, N_2 and N_3 can be determined as a function of time using the Laplace transform as follow:

$$\frac{dN_1}{dt} = -\lambda_1 N_1 \qquad (1-4)$$

$$\mathcal{L}\left(\frac{dN_1}{dt}\right) + \mathcal{L}(\lambda_1 N_1) = \mathcal{L}(0)$$

$$sN_1(s) - N_1(0) + \lambda_1 N_1(s) = 0$$

$$N_1(s)(s+\lambda_1) = N_1(0)$$

$$N_1(s) = \frac{N_1(0)}{(s+\lambda_1)} \qquad (i)$$

$$\mathcal{L}^{-1}[N_1(s)] = \mathcal{L}^{-1}\left[\frac{N_1(0)}{(s+\lambda_1)}\right]$$

$$N_1(t) = N_1(0)e^{-\lambda_1 t}$$

$$\frac{dN_2}{dt} = \lambda_1 N_1 - \lambda_2 N_2 \qquad (1-5)$$

$$\mathcal{L}\left(\frac{dN_2}{dt} + \lambda_2 N_2\right) = \mathcal{L}(\lambda_1 N_1)$$

$$\mathcal{L}\left(\frac{dN_2}{dt}\right) + \mathcal{L}(\lambda_2 N_2) = \mathcal{L}(\lambda_1 N_1)$$

$$sN_2(s) - N_2(0) + \lambda_2 N_2(s) = \lambda_1 N_1(s) \qquad (ii)$$

Substituting from (i) in to (ii) we get

$$sN_2(s) + \lambda_2 N_2(s) = \lambda_1 \frac{N_1(0)}{(s+\lambda_1)} \qquad since\ N_2(0) = 0$$

$$(s+\lambda_2)N_2(s) = \lambda_1 \frac{N_1(0)}{(s+\lambda_1)}$$

$$N_2(s) = \lambda_1 \frac{N_1(0)}{(s+\lambda_1)(s+\lambda_2)} \qquad (iii)$$

$$\mathcal{L}^{-1}[N_2(s)] = \lambda_1 N_1(0)\mathcal{L}^{-1}\left[\frac{1}{(s+\lambda_1)(s+\lambda_2)}\right]$$

$$\mathrm{N}_2(t)=\frac{\lambda_1}{\lambda_2-\lambda_1}N_1(0)\mathcal{L}^{-1}\left[\frac{1}{(s+\lambda_1)}-\frac{1}{(s+\lambda_2)}\right]$$

$$\mathrm{N}_2(t)=\frac{\lambda_1}{\lambda_2-\lambda_1}N_1(0)\left[e^{-\lambda_1 t}-e^{-\lambda_2 t}\right]$$

$$\frac{\mathrm{dN}_3}{\mathrm{dt}}=-\lambda_2\mathrm{N}_2 \qquad (1-6)$$

$$\mathcal{L}\left[\frac{\mathrm{dN}_3}{\mathrm{dt}}\right]=-\lambda_2\mathcal{L}[\mathrm{N}_2]$$

$$\boldsymbol{s}\mathrm{N}_3(\boldsymbol{s})-\mathrm{N}_3(0)=-\lambda_2\mathrm{N}_2(\boldsymbol{s}) \qquad (iv)$$

Substituting from (iii) in to (iv) we get

$$\boldsymbol{s}\mathrm{N}_3(\boldsymbol{s})=-\lambda_2\lambda_1\frac{N_1(0)}{(\boldsymbol{s}+\lambda_1)(\boldsymbol{s}+\lambda_2)} \quad ,since\ \ \mathrm{N}_3(0)=0$$

$$\mathrm{N}_3(\boldsymbol{s})=\lambda_2\lambda_1\frac{N_1(0)}{\boldsymbol{s}(\boldsymbol{s}+\lambda_1)(\boldsymbol{s}+\lambda_2)}$$

$$\mathcal{L}^{-1}[\mathrm{N}_3(\boldsymbol{s})]=\lambda_2\lambda_1N_1(0)\mathcal{L}^{-1}\left[\frac{1}{\boldsymbol{s}(\boldsymbol{s}+\lambda_1)(\boldsymbol{s}+\lambda_2)}\right]$$

$$\mathrm{N}_3(t)=\lambda_2\lambda_1N_1(0)\mathcal{L}^{-1}\left[\frac{1}{\lambda_1\lambda_2}\frac{1}{\boldsymbol{s}}-\frac{1}{\lambda_1(\lambda_2-\lambda_1)}\frac{1}{(\boldsymbol{s}+\lambda_1)}+\frac{1}{\lambda_2(\lambda_2-\lambda_1)}\frac{1}{(\boldsymbol{s}+\lambda_2)}\right]$$

$$\mathrm{N}_3(\boldsymbol{t})=N_1(0)\left[1-\frac{\lambda_2}{(\lambda_2-\lambda_1)}e^{-\lambda_1 t}+\frac{\lambda_1}{(\lambda_2-\lambda_1)}e^{-\lambda_2 t}\right]$$

3. CONCLUSION

Through this paper we present the applications of Laplace Transform in nuclear physics field especially in decay and growth of nucleus or atoms. Laplace transform is a very effective tool to simplify many complex problems in nuclear physics field.

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